HEALTH INVESTMENT AND ECONOMIC DEVELOPMENT

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Abstract

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A great deal of research has examined the relationships between economic growth and measures of health and longevity. In this paper I model these interactions dynamically by adapting the familiar Solow model to incorporate various health-related factors, such as investment in medical R&D, population growth due to increased life expectancy, and knowledge about health. I demonstrate that each of these models possesses a unique non-trivial steady-state which, under suitable conditions, is stable. I further analyze each model by considering comparative statics for the steady-state with respect to the various parameters in the model, and interpret these economically. My models suggest that both economic growth and life expectancy are affected by R&D-related factors, and that knowledge about health care is a valuable concept that deserves inclusion into a model designed to study these interactions. I further find that accounting for population growth produces a model which is both more well-behaved and which displays consistency with recent literature that concludes increased longevity in the population has a negative influence on per capita economic growth.

1. Introduction

This paper studies the relationship between economic growth and life expectancy. I am particularly interested in considering this relationship for the economies of developing countries, and especially in regions such as Sub-Saharan Africa, which tend to perform poorly in measures of health and economic strength.

The World Health Organization's 1996 edition of the World Health Report displays a clear inverse relationship between level of economic development and various measures of child mortality ([18], p. 14). In particular, in 1995 the child mortality rate (probability of dying before the age of five) was only 8.5 per 1000 in industrialized nations with developed market economies, but increases to 155.5 per 1000 in the least developed countries. Furthermore, such countries are largely African: of the 24 nations with a child mortality rate in excess of 100 per 1000, 17 were located in the African continent. Similarly, the World Bank's 2008 World Development Indicators report ([16], pp. 4-5) finds that although economic growth in Sub-Saharan Africa is improving, there is still a huge disparity in economic performance (measured in GDP per capita) between Sub-Saharan Africa and all other regions except for South Asia, which performs better in some formulations and worse in others. In particular, Sub-Saharan Africa is the worst performing region in terms of PPP (purchasing power parity), and has the widest income disparity (the richest 20% earn 64.5% of the income, while the poorest 20% earn only 3.6%).

I take observations from Cutler, Deaton, and Lleras-Muney [3] on the "three phases" of mortality reduction as a conceptual starting point for my models. Briefly, the first phase encompasses the middle of the 18th century to the middle of the 19th century, where the causes of health improvements are still debated, although factors such as improvements in nutrition and basic public health measures could very likely have played an important part.

I will not explicitly discuss this phase any further in what is to follow, preferring to concentrate on the latter two. The second phase, which I outline in more depth below, consisted of the final decades of the 19th century and the initial decades of the 20th century until about the time of the Great Depression, and can be roughly described as the era of public health improvements. The third phase, which is also discussed in more detail momentarily, is the modern period from the 1930s onward to present day, and can be described as the era of "big medicine".

Working backwards, I begin by considering the third phase, where as previously mentioned, modern medicine has been the biggest catalyst in reductions of mortality. This period is characterized by medical innovations involving considerable research and development (R&D), from simple drug treatments such as antibiotics and vaccines to complex and costly personal treatments involving designer drugs, advanced technology, and complicated procedures at state-of-the-art health care facilities. Considerable research has been done on the rate of the return pharmaceutical companies receive from investments in R&D, with disconcerting results. The work of Dimasi et. al. ([5]) showed that in the specific case of the potential prescription drugs which reached phase one clinical trials (not an easy task to begin with), fewer than a quarter were eventually approved by the Food and Drug Administration for consumer use. Grabowski and Vernon ([8],[9]) later went on to show that during the 1970s and 1980s, approximately 70% of new drugs did not provide positive returns to investment, i.e. the cost of R&D exceed the profits generated from sales. Consequently, it has been observed that in an effort to offset these losses, pharmaceutical companies tend to prefer drug treatments over vaccines ([10]) and favor drugs aimed at customers in affluent developed countries, a situation which is hardly beneficial to poorer developing ones. It seems crucial therefore that any model we attempt to use to study the dynamics of economic growth and life expectancy in a developing country takes into account both the amount of investment which goes towards R&D, and the probability that R&D is actually effective.

Of course, it is unreasonable to assume that the only health care innovations which will affect life expectancy are those related to pharmaceutical drugs and other highly research-intensive treatment methods. Returning again to the discussion of phases in [3], I consider the second phase that occurred at the turn of the century, where improved health practices related to simple things such as personal hygiene and use of clean water influenced both levels of mortality and economic growth. Considerable research has been done on the effect personal behavior and public health advances had on the decline of mortality rates in the early twentieth century United States; see for example [6] and [4]. Ewbank and Preston ([6]) also note that these results may be the closest analog to the current situation for many developing countries. Preston ([11], [12]) further argues that economic growth cannot be the only reason for increased life expectancy, observing that given a level of income from a now-developed country at some point in the past, and given a developing country whose income corresponds to the given one today, the life expectancy of the developing country will almost certainly be higher than the life expectancy of the developed country was at the time when their respective incomes were actually equal. With this in mind, I expand my model to study the interactions between physical capital, economic growth, and knowledge about health care. By "knowledge about health care" I mean any information, behavioral change, or general health care improvement that comes with little or no direct cost to the individual (although some, such as the knowledge that treating drinking water will reduce the risk of many diseases, often require some degree of funding to implement).

The recent work of Acemoglu and Johnson ([1]) has challenged the popular viewpoint that increases in life expectancy will have a significant positive impact on economic growth. Their research considers the modern period that corresponds roughly to the third phase discussed above, which they term the "international epidemiological transition", so its conclusions are particularly relevant to the models I am considering. Among their many results and conclusions, perhaps the most interesting is that countries displaying large increases in life expectancy also experience relative declines in GDP per capita. As explained by the authors,

there is a natural explanation for this phenomenon using classical growth theory, namely that an increase in longevity results in a corresponding increase in total population, and this leads to an initial decrease in income per capita. In the classical theory one would expect to see an increase in output after some time has elapsed, as the workforce expands in size and capital accumulation grows. However, this is under the assumption that increased life expectancy significantly improves productivity. The presence of limiting factors on the benefits of improved life expectancy could stunt this expected increase, leading to the negative growth results observed in [1]. With this in mind, I generalize the model initially discussed to account for a negative effect of longevity on physical capital.

My models support the following conclusions. First of all, improvements in R&D (specifically, increased investment and increased success probability) have positive influences on both life expectancy and economic growth (the latter relationship being the less obvious and hence more interesting result). Next, knowledge about health care should be taken into consideration when studying the interactions between economic growth and health. Its inclusion makes sense in light of existing literature and because it incorporates factors which are not accounted for by the other parameters. Moreover, including health knowledge improves the theoretical model since it eliminates a paradoxical result from the basic model. Finally, the addition of the population growth factor gives results on the relationship between economic growth and population growth more consistent with [1].

2. The Basic Model

I formulate a model to study the dynamics between economic growth and health based on the familiar Solow growth model (see for example [14] and [17]). Let $K = K(t)$ denote capital at time t, $L = L(t)$ denote labor at time t, and A denote productivity, which I assume to be constant. (where by productivity I mean that for fixed capital and labor amounts, a higher

value of A results in higher output). Throughout this model, I measure production with the standard Cobb-Douglas production function

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$$

where $0 \leq \alpha \leq 1$. Dividing by the quantity of labor and using constant returns to scale yields the corresponding formula for per worker production

$$y = \frac{Y}{L} = F(K/L, 1) = Ak^{\alpha}$$

where $k = k(t) = \frac{K}{L}$ measures physical capital per worker at time t (henceforth, I frequently omit the "per worker" qualifier since I only consider physical capital measured in these terms). My goal is to determine how physical capital changes over time. Working from the Solow model, I expect that this change is described roughly by

$$\dot{k} = \gamma_k y - \delta k$$

where γ_k denotes the investment rate for physical capital, and δ is the rate of depreciation for existing capital. In other words, the change in physical capital is determined by how much of total output is re-invested less how much existing capital depreciates in value. I incorporate life expectancy into this model as follows. Let $\phi = \phi(t)$ denote life expectancy at time t, in the sense that ϕ is the probability of a worker living at time t. I assume that if people have an increased probability of living for a longer period of time, they will invest more income now to pay for their increased future needs later. This tells me that the effective investment rate is now $\gamma_k \phi$ rather than simply γ_k. This leads to the modified equation

$$\dot{k} = \gamma_k \phi A k^{\alpha} - \delta k$$

I provide a careful mathematical justification for this formula, starting from the aggregate variables and ending with the per worker equation, in the Appendix. Now, I do not simply wish to consider life expectancy as a parameter in the growth equation for physical capital,

but instead study how it changes over time and how it interacts with economic growth. I first identify two factors which I feel should have an effect on life expectancy and whose inclusion as parameters is motivated by the discussion of research and development in the introduction. Namely, these are the investment rate in R&D, and the probability that R&D is effective. Higher probability of effective R&D and higher total investment of output into R&D (given by multiplying the investment rate by the total output) should improve life expectancy. However, I cannot expect growth in life expectancy to remain positive forever; a total life expectancy for a country above 100 years does not seem realistic, and even this is a generous upper bound. This leads me to the growth equation

$$\dot{\phi} = \pi\gamma_\phi y\phi - \sigma\phi^2$$

where π denotes the probability that R&D is effective, γ_ϕ denotes the investment rate for R&D, and σ is an undetermined damping constant reflecting the aforementioned fact that the growth rate of life expectancy must eventually slow down (an actual value of σ would presumably come from some sort of real-life data). Writing $\gamma = \gamma_k + \gamma_\phi$ and using $\gamma - \gamma_k$ in place of γ_ϕ, I may write the growth equation as

$$\dot{\phi} = \pi\phi(\gamma - \gamma_k)Ak^\alpha - \sigma\phi^2$$

Thus, my model for the dynamics between economic growth and health is built around the interactions of physical capital and life expectancy, given by the following system of non-linear autonomous differential equations:

$$\dot{k} = \phi\gamma_k Ak^\alpha - \delta k$$

$$\dot{\phi} = \pi\phi(\gamma - \gamma_k)Ak^\alpha - \sigma\phi^2$$

For these two equations to describe the economy, I must ensure that the model is reasonably well-behaved. Mathematically, this is ensured by the presence of a stable steady-state

solution for the system. What this means is that for the initial economic state $k(0) = k_0$, $\phi(0) = \phi_0$ (where the values k_0 and ϕ_0 are determined by historical economic and demographic data), over time the steady-states for capital, life expectancy, and output per worker will converge to this stable steady-state solution. We determine the steady-states of the system (k^*, ϕ^*) by setting both equations equal to zero and solving simultaneously. The homogeneous system can be written as

$$\phi^* \gamma_k A (k^*)^\alpha = \delta k^*$$

$$\pi \phi^* (\gamma - \gamma_k) A (k^*)^\alpha = \sigma (\phi^*)^2$$

Dividing the first equation by the second one gives

$$\frac{\gamma_k}{\pi(\gamma - \gamma_k)} = \frac{\delta k^*}{\sigma(\phi^*)^2}$$

which can be solved for k^* as

$$k^* = \frac{\gamma_k \sigma (\phi^*)^2}{\pi(\gamma - \gamma_k)\delta}$$

Substituting this expression for k^* into the second equation gives

$$\begin{aligned}
\sigma(\phi^*)^2 &= \pi \phi^* (\gamma - \gamma_k) A \left(\frac{\gamma_k \sigma (\phi^*)^2}{\pi(\gamma - \gamma_k)\delta} \right)^\alpha \\
&= \frac{\pi \phi^* (\gamma - \gamma_k) A \gamma_k^\alpha \sigma^\alpha (\phi^*)^\alpha}{\pi^\alpha (\gamma - \gamma_k)^\alpha \delta^\alpha} \\
&= \pi^{1-\alpha} (\phi^*)^{2-\alpha} (\gamma - \gamma_k)^{1-\alpha} A \gamma_k^\alpha \sigma^\alpha \delta^{-\alpha}
\end{aligned}$$

Dividing by $\sigma(\phi^*)^{2-\alpha}$ on each side of this equation gives

$$(\phi^*)^\alpha = \frac{A \pi^{1-\alpha} \gamma_k^\alpha (\gamma - \gamma_k)^{1-\alpha}}{\delta^\alpha \sigma^{1-\alpha}}$$

which yields the steady-state value for ϕ^*:

$$\phi^* = \frac{A^{1/\alpha}\gamma_k\sigma^{1-1/\alpha}}{\pi^{1-1/\alpha}\delta(\gamma-\gamma_k)^{1-1/\alpha}}$$

Now substituting ϕ^* into the equation for k^* gives the steady-state value for k^*:

$$k^* = \frac{A^{2/\alpha}\gamma_k^3\sigma^{3-2/\alpha}}{\pi^{3-2/\alpha}\delta^3(\gamma-\gamma_k)^{3-2/\alpha}}$$

Thus, the system has a single steady-state (k^*, ϕ^*), given by

$$(k^*, \phi^*) = \left(\frac{A^{2/\alpha}\gamma_k^3\sigma^{3-2/\alpha}}{\pi^{3-2/\alpha}\delta^3(\gamma-\gamma_k)^{3-2/\alpha}}, \frac{A^{1/\alpha}\gamma_k\sigma^{1-1/\alpha}}{\pi^{1-1/\alpha}\delta(\gamma-\gamma_k)^{1-1/\alpha}}\right)$$

Next, I demonstrate that (k^*, ϕ^*) is a stable steady-state for the system in the case that $\alpha = \frac{1}{3}$. Choosing a specific value of α here allows us me give an affirmative answer to the stability question without making any other explicit choices for the values of the exogenous parameters. The choice of $\alpha = \frac{1}{3}$ is not made arbitrarily. A measurement of capital's share of income (essentially, the fraction of national income that is paid out as rent on capital, which can be shown to equal α by considering the marginal product of capital for our production function) for a large sample of countries results in a mean value of $\alpha \approx 0.35$, which is close to $\frac{1}{3}$. Moreover, capital's share of income does not seem to have a discernable relationship to GDP per capita. For instance, there are developing countries which exceed the mean (such as Botswana, where capital's share of income is about 0.55) and ones which are well below the mean (such as Sri Lanka, where it is approximately 0.2). As such, it seems reasonable to use the value $\alpha = \frac{1}{3}$ without too much worry that it is only a good estimation for developed countries. In fact, one possibility is that capital's share of income is identical for all countries and the variations in estimates are due to measurement error and insufficiency of the data being used. For a more detailed treatment of factor shares and the quoted study, see Weil ([17], pp. 55-57).

Now we set $\dot{k} = F(k, \phi)$ and $\dot{\phi} = G(k, \phi)$, and write

$$F(k, \phi) = \phi\gamma_k A k^{1/3} - \delta k$$

$$G(k, \phi) = \pi\phi(\gamma - \gamma_k) A k^{1/3} - \sigma\phi^2$$

from which we obtain the partial derivative formulas

$$F_k = \tfrac{1}{3}\phi\gamma_k A k^{-2/3} - \delta \qquad F_\phi = \gamma_k A k^{1/3}$$

$$G_k = \tfrac{1}{3}\pi\phi(\gamma - \gamma_k) A k^{-2/3} \quad G_\phi = \pi(\gamma - \gamma_k) A k^{1/3} - 2\sigma\phi$$

Evaluating these at the steady-state (k^*, ϕ^*) gives the Jacobian matrix

$$J = \begin{bmatrix} F_k(k^*, \phi^*) & F_\phi(k^*, \phi^*) \\ G_k(k^*, \phi^*) & G_\phi(k^*, \phi^*) \end{bmatrix} = \begin{bmatrix} -\frac{2\delta}{3} & \frac{A^3\pi\gamma_k^2(\gamma-\gamma_k)}{\delta\sigma} \\ \frac{\pi(\gamma-\gamma_k)\delta}{3\gamma_k} & -\frac{A^3\pi^2\gamma_k(\gamma-\gamma_k)^2}{\delta\sigma} \end{bmatrix}$$

The characteristic polynomial for J is

$$\det(J - \lambda I) = \lambda^2 + \left(\frac{2\delta}{3} + \frac{3A^3\pi^2\gamma_k(\gamma - \gamma_k)^2}{3\sigma\delta}\right)\lambda + \frac{A^3\pi^2\gamma_k(\gamma - \gamma_k)^2}{3\sigma}$$

Letting B denote the constant term in the characteristic polynomial, it follows that

$$\lambda^2 + \left(\frac{2\delta}{3} + \frac{3B}{\delta}\right)\lambda + B = 0$$

This equation has solutions

$$\lambda = \frac{-(2\delta^2 + 9B) \pm \sqrt{4\delta^4 + 81B^2}}{6\delta}$$

Now $\delta > 0$ and $B > 0$, and also $\sqrt{4\delta^4 + 81B^2} < 2\delta^2 + 9B$, so that $-(2\delta^2 + 9B) \pm \sqrt{4\delta^4 + 81B^2} < 0$. It follows that both eigenvalues of the Jacobian are real and negative. This implies that (k^*, ϕ^*) is an asymptotically stable steady-state of the system using Theorem 25.5 of [15].

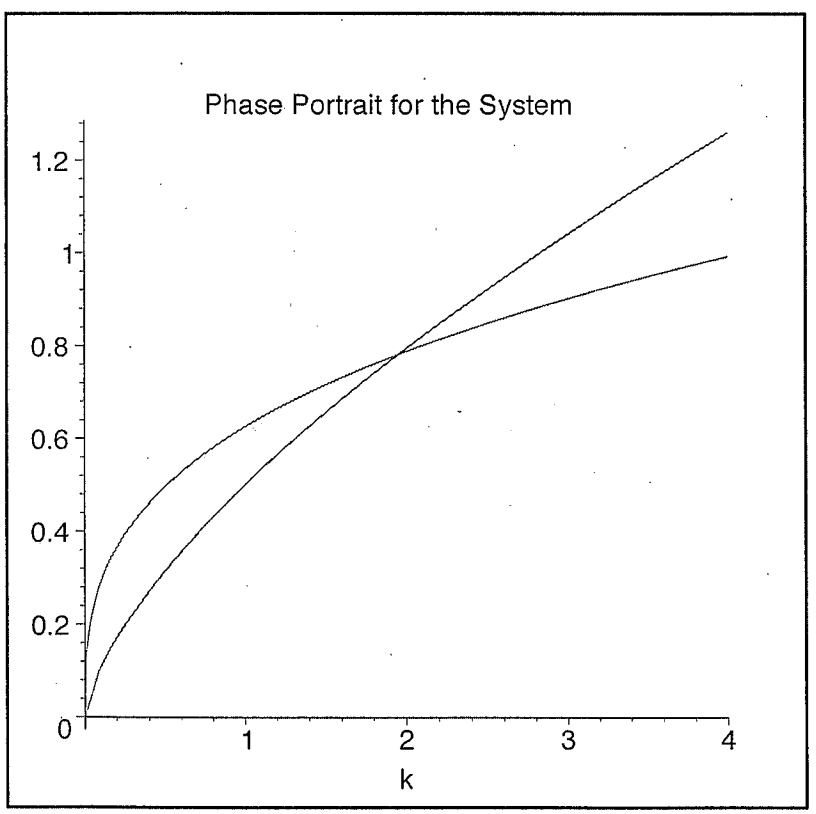
It is now straightforward to determine the phase-portrait of the system by sketching the $\dot{k} = 0$ and $\dot{\phi} = 0$ loci. Again working in the case $\alpha = \frac{1}{3}$, setting $\dot{k} = 0$ implies that

$$k^{1/3}(\phi\gamma_k A - \delta k^{2/3}) = 0$$

which gives locus equations $k = 0$ (the ϕ-axis) and $\phi = \frac{\delta}{A\gamma_k}k^{2/3}$ (for $k \neq 0$). Horizontal isoclines occur when $\dot{\phi} = 0$ and $\dot{k} \neq 0$. Setting $\dot{\phi} = 0$ implies that

$$\phi(\pi(\gamma - \gamma_k)Ak^{1/3} - \sigma\phi) = 0$$

which gives locus equations $\phi = 0$ (the k-axis) and $\phi = \frac{\pi A(\gamma-\gamma_k)}{\sigma}k^{1/3}$. I obtain the following phase-portrait for the system:



Note that the non-trivial $\dot{k} = 0$ locus is the curve which begins lower on the graph and ends up above the non-trivial $\dot{\phi} = 0$ locus. Returning to the general case, I have previously

calculated that

$$k^* = \frac{A^{2/\alpha}\gamma_k^3\sigma^{3-2/\alpha}}{\pi^{3-2/\alpha}\delta^3(\gamma-\gamma_k)^{3-2/\alpha}}, \qquad \phi^* = \frac{A^{1/\alpha}\gamma_k\sigma^{1-1/\alpha}}{\pi^{1-1/\alpha}\delta(\gamma-\gamma_k)^{1-1/\alpha}}$$

Since (k^*, ϕ^*) is an asymptotically stable steady-state, substituting k^* into the output per worker formula gives the corresponding steady-state output per worker:

$$y^{ss} = A\left(\frac{A^{2/\alpha}\gamma_k^3\sigma^{3-2/\alpha}}{\pi^{3-2/\alpha}\delta^3(\gamma-\gamma_k)^{3-2/\alpha}}\right)^{\alpha} = \frac{A^3\pi^{2-3\alpha}\gamma_k^{3\alpha}(\gamma-\gamma_k)^{2-3\alpha}}{\delta^{3\alpha}\sigma^{2-3\alpha}}$$

Now I consider some comparative statics; that is, partial derivatives of k^* and ϕ^* with respect to some of the parameters. These allow me to study how the steady-states for capital and life expectancy change as the various parameters change individually. Specifically, I say that a variable and a parameter exhibit a positive relationship if an increase in that parameter results in an increase to the steady-state value for that variable, with an analogous definition for negative relationships. These correspond to the associated comparative static being either positive or negative. Explicit formulas for the comparative statics are given in the Appendix.

My first observation based on comparative statics is that both physical capital and life expectancy display a positive relationship with productivity, so that an increase in productivity shifts both steady-state values positively. This relationship is consistent with some of the results of Pritchett and Summers [13] concerning the effect of increased economic output on life expectancy. Specifically, their ordinary least squares estimates imply that a 10% increase in income results in a modest increase in mean life expectancy (about one month), although the imprecision of the estimates results in a coefficient for the statistic that is significant at the 10% level but not at the 5% level. Further, life expectancy also has a positive relationship with the probability that R&D is successful, and with the rate of investment in health care. Both of these results would seem to agree with intuition. A higher probability of successful R&D would lead to more health-improving medicines and treatments reaching the general population, which should lead to an increase in longevity. Similarly, increased investment in

R&D should also eventually result in health-care advances that will prove beneficial to life expectancy.

The other comparative statics cannot be analyzed so simply, as the formulas depend on α in such a way that the relationships they describe are sometimes positive and sometimes negative. However, by recalling what α actually represents, I can simplify the discussion of these cases considerably. Recall that α describes capital's share of income, and that empirical evidence ([17], pp. 56-57) suggests the mean value for the world's countries to be approximately 0.35, which is only slightly different than $\frac{1}{3}$. Further, the range of observed values lies roughly between 0.2 and 0.6. For my current purposes, most of the changes in the behavior of the comparative statics I have not yet discussed occur at the value $\alpha = \frac{2}{3}$. Based on the observed data, values of α at or greater than this value do not seem all that likely, so I should feel reasonably confident in assuming that $\alpha < \frac{2}{3}$ here. If this is the case, I observe positive relationships between physical capital and the R&D-related parameters. One possible explanation for this behavior is that as R&D increases both in quantity and in quality, workers live longer and healthier lives, and hence are more productive over the years in which they are physically able to work, resulting in an increase in capital.

Unfortunately, not all the comparative statics give results that are as intuitive and straightforward to explain as the ones I have presented so far. Concerning the relationship between life expectancy and investment in capital, I observe a somewhat complicated behavior that depends on both the value of α and the relationship between the investment rates for physical capital and for R&D. Roughly speaking, I will see a positive relationship between life expectancy and investment in physical capital so long as investment in capital does not become too large relative to investment in R&D. Specifically, for my preferred case of $\alpha = \frac{1}{3}$, the relationship in question will be positive if and only if investment in capital is less than half as much as investment in R&D. More troubling is the comparative statics result for the relationship between capital and investment in physical capital; namely, it is negative (for

the situation we are considering, where $\alpha < \frac{2}{3}$). Even though I am looking at a situation where capital's share of income is less than labor's, increasing investment in capital seems like it should result in an increase to the steady-state for physical capital.

The comparative statics can thus be seen to support the following conclusions. First, capital and life expectancy exhibit positive relationships with the exogenous parameters in many of the situations where I would expect them to. Second, the ones that change seem to mostly give sensible relationships in the situation which I usually restrict to (where capital's share of income is estimated to be $\frac{1}{3}$), with one obvious exception. An especially noteworthy result is that in my model, increased probability that R&D is successful and higher investment rates in R&D both result in an increase to the steady-state physical capital, in addition to the expected increases to life expectancy. These observations suggest that increasing the rate at which capital is invested into R&D is a sound economic and social policy, as it should lead to both increased wealth and increased health. Similarly, there is an economic incentive to increase the likelihood that medical R&D proves successful, as this should stimulate economic growth. In summary, my model produces results which are at least partly consistent with existing literature, and which appear to be mostly economically sensible. However, the paradoxical result of a negative relationship between physical capital and investment in physical capital is a serious defect in the model, whose presence suggests that some degree of modification is necessary.

3. Extending the Model to Include Knowledge About Health

Having developed a reasonable, if partly flawed, model to study the dynamics of physical capital k and life expectancy ϕ, I seek to extend this model to include the effects of knowledge about health, which I denote by h. There are two reasons I wish to consider this extension. First, as discussed in the introduction, the work of Ewbanks and Preston ([6]) suggests that

in the early twentieth century, behavioral changes in America with regards to personal health care played a significant role in the observed declines in infant mortality. It should therefore be reasonable to expect these same sorts of behavioral changes, many of which are related to improved knowledge about personal heath care, would result in a positive change in life expectancy for developing countries today. Second, I take notice of the fact that for many developing countries, investing large amounts of capital into R&D may not be practical, as they may not even have enough income to invest at all. In light of this, it seems very likely that knowledge-based improvements in health care may have a more noticeable effect on both life expectancy and economic growth, due to both their effectiveness and lack of serious cost (at least when compared to R&D, and even treatment).

I thus seek to incorporate knowledge about health care into my model as a variable, so that I can consider both how such knowledge grows and how it affects capital and life expectancy. First of all, I assume that knowledge about health care increases as output increases. Thinking again about the work of Pritchett and Summers ([13]) and their conclusions regarding the effects of economic output on health, it is not hard to imagine reasons why I might expect this to happen. Knowledge about health care could be more easily disseminated to the public if they have the means to understand and receive it, and improved economic output could result in better literacy and methods of communication. However, knowledge about health care is clearly a limited quantity, so I cannot expect growth to increase indefinitely. Also, I should expect that knowledge about health care depreciates over time in some way, as information that was once new and important becomes obsolete. These considerations lead us to the equation

$$\dot{h} = Dk^{\alpha} - \beta h$$

where D is some exogenous parameter which determines by how much output affects knowledge about health care, and β is a damping constant that reflects the fact that knowledge about health care is limited and accumulation of new knowledge becomes more difficult as

the existing bank of knowledge becomes more extensive. Now, I consider the effect that knowledge about health care has on physical capital and life expectancy. Returning again to the Solow model, I consider the case where the production function takes the form

$$Y = F(K, hL) = AK^{\alpha}h^{1-\alpha}L^{1-\alpha}$$

where $K = K(t)$ and $L = L(t)$ measure capital and labor at time t as before, A is once again production, and $h = h(t)$ measures knowledge about health care (see Romer [14], §1.2 for a similar development). Effectively what I am doing in this model is assuming that knowledge about health care affects output because of its positive effect on the labor force (since healthier workers should improve output). Now I divide by hL to obtain the output formula in per "healthy worker" terms, which is

$$y = Ak^{\alpha}$$

where $k = K/hL$ denotes physical capital per healthy worker at time t. I eventually obtain (see the Appendix for the details), a similar equation to the equation for physical capital from the previous equation, but including the growth of knowledge about health care

$$\dot{k} = \gamma_k \phi A k^{\alpha} - k(\dot{h} + \delta)$$

with γ_k the investment rate in physical capital and δ the depreciation rate. For life expectancy, the appropriate equation is

$$\pi\phi(\gamma - \gamma_k)Ah^{1-\alpha}k^{\alpha} - \sigma\phi^2$$

with $\gamma = \gamma_k + \gamma_\phi$ (where γ_ϕ is the investment rate in R&D), π the probability that R&D is successful, and σ is the damping constant described earlier. Hence for my new model I am

considering the three-dimensional system of differential equations given by

$$\dot{k} = \phi\gamma_k A k^\alpha - k(\dot{h} + \delta)$$

$$\dot{\phi} = \pi\phi(\gamma - \gamma_k)Ah^{1-\alpha}k^\alpha - \sigma\phi^2$$

$$\dot{h} = Dk^\alpha - \beta h$$

with some initial condition $k(0) = k_0, \phi(0) = \phi_0, h(0) = h_0$ describing the initial state of the economy and of health.

Steady-state solutions (k^*, ϕ^*, h^*) occur when the system is homogeneous:

(1) $$\phi^*\gamma_k A(k^*)^\alpha = \delta k^*$$

(2) $$\pi\phi^*(\gamma - \gamma_k)A(h^*)^{1-\alpha}(k^*)^\alpha = \sigma(\phi^*)^2$$

(3) $$D(k^*)^\alpha = \beta h^*$$

Solving (3) for $(k^*)^\alpha$ gives

(4) $$(k^*)^\alpha = \frac{\beta h^*}{D}$$

Combining (1) and (4) gives

(5) $$k^* = \frac{A\gamma_k\beta}{D\delta}h^*\phi^*$$

Similarly, (2) and (4) give

(6) $$\phi^* = \frac{A\pi(\gamma - \gamma_k)\beta}{D\sigma}(h^*)^{2-\alpha}$$

Substituting (6) into (5) yields the further reduction

(7) $$k^* = \frac{A^2\gamma_k(\gamma - \gamma_k)\beta^2\pi}{D^2\delta\sigma}(h^*)^{3-\alpha}$$

Raising (7) to the power α and substituting (4) into the left-hand side gives

$$\frac{\beta h^*}{D} = \frac{A^{2\alpha}\gamma_k^\alpha(\gamma-\gamma_k)^\alpha\pi^\alpha\beta^{2\alpha}}{D^{2\alpha}\delta^\alpha\sigma^\alpha}(h^*)^{3\alpha-\alpha^2} \tag{8}$$

which can be solved for h as

$$(h^*)^{\alpha^2-3\alpha+1} = \frac{A^{2\alpha}\gamma_k^\alpha(\gamma-\gamma_k)^\alpha\pi^\alpha\beta^{2\alpha-1}}{D^{2\alpha-1}\delta^\alpha\sigma^\alpha} \tag{9}$$

If $\alpha \neq \frac{1}{2}(3-\sqrt{5})$, then $(\alpha^2-3\alpha+1)^{-1}$ exists, and so I can write

$$h^* = \left(\frac{A^2\gamma_k(\gamma-\gamma_k)\pi\beta^{2-1/\alpha}}{D^{2-1/\alpha}\delta\sigma}\right)^{\frac{\alpha}{\alpha^2-3\alpha+1}} \tag{10}$$

Now combining equations (6) and (10) gives

$$\phi^* = \frac{A\pi(\gamma-\gamma_k)\beta}{D}\left(\frac{A^2\gamma_k(\gamma-\gamma_k)\pi\beta^{2-1/\alpha}}{D^{2-1/\alpha}\delta\sigma}\right)^{\frac{\alpha(2-\alpha)}{\alpha^2-3\alpha+1}} \tag{11}$$

which can be algebraically simplified to

$$\phi^* = \left(\frac{A^{1+\alpha-\alpha^2}\pi^{1-\alpha}(\gamma-\gamma_k)^{1-\alpha}\gamma_k^{\alpha(2-\alpha)}D^{(1-\alpha)^2}}{\beta^{(1-\alpha)^2}\delta^{1-\alpha}\sigma^{1-\alpha}}\right)^{\frac{1}{\alpha^2-3\alpha+1}} \tag{12}$$

Finally, combining equations (7) and (10) gives

$$k^* = \frac{A^2\gamma_k(\gamma-\gamma_k)\beta^2\pi}{D^2\delta\sigma}\left(\frac{A^2\gamma_k(\gamma-\gamma_k)\beta^2\pi}{D^{2-1/\alpha}\delta\sigma}\right)^{\frac{\alpha(3-\alpha)}{\alpha^2-3\alpha+1}} \tag{13}$$

which simplifies to

$$k^* = \left(\frac{A^2\gamma_k(\gamma-\gamma_k)\pi\beta^{\alpha-1}}{D^{\alpha-1}\delta\sigma}\right)^{\frac{1}{\alpha^2-3\alpha+1}} \tag{14}$$

So I get a unique steady-state (k^*, ϕ^*, h^*) for the system, given by

$$\left(\frac{A^2\gamma_k(\gamma-\gamma_k)\pi\beta^{\alpha-1}}{D^{\alpha-1}\delta\sigma}, \frac{\pi^{1-\alpha}(\gamma-\gamma_k)^{1-\alpha}\gamma_k^{\alpha(2-\alpha)}D^{(1-\alpha)^2}}{A^{\alpha^2-\alpha-1}\beta^{(1-\alpha)^2}\delta^{\alpha(2-\alpha)}\sigma^{1-\alpha}}, \left(\frac{A^2\gamma_k(\gamma-\gamma_k)\pi\beta^{2-1/\alpha}}{D^{2-1/\alpha}\delta\sigma}\right)^\alpha\right)^{\frac{1}{\alpha^2-3\alpha+1}}$$

and the associated steady-state output per worker is

$$y_{ss} = \left(\frac{A^{\alpha^2-\alpha+1}\gamma_k^{\alpha}(\gamma-\gamma_k)^{\alpha}\pi^{\alpha}\beta^{\alpha^2-\alpha}}{D^{\alpha^2-\alpha}\delta^{\alpha}\sigma^{\alpha}} \right)^{\frac{1}{\alpha^2-3\alpha+1}}$$

Determining whether or not the point is a stable steady-state for the system is much more complicated in this case since the characteristic equation resulting from the Jacobian matrix will be a cubic. Again, I simplify the calculations by assuming that $\alpha = \frac{1}{3}$ (the justification for this choice being the same as in the previous model). In this case, I obtain the values

$$k^* = \frac{A^{18}\gamma_k^9(\gamma-\gamma_k)^9\pi^9 D^6}{\beta^6\delta^9\sigma^9}$$

$$\phi^* = \frac{A^{11}\pi^6\gamma_k^5(\gamma-\gamma_k)^6 D^4}{\beta^4\delta^5\sigma^6}$$

$$h^* = \frac{A^6\gamma_k^3(\gamma-\gamma_k)^3\pi^3 D^3}{\beta^3\delta^3\sigma^3}$$

Setting $\dot{k} = F(k,\phi,h)$, $\dot{\phi} = G(k,\phi,h)$ and $\dot{h} = H(k,\phi,h)$ we write

$$F = F(k,\phi,h) = A\gamma_k\phi k^{1/3} - k(Dk^{1/3} - \beta h + \delta)$$

$$G = G(k,\phi,h) = A(\gamma-\gamma_k)\pi\phi h^{2/3}k^{1/3} - \sigma\phi^2$$

$$H = H(k,\phi,h) = Dk^{1/3} - \beta h$$

I obtain the Jacobian matrix of partial derivatives

$$J = \begin{bmatrix} F_k & F_\phi & F_h \\ G_k & G_\phi & G_h \\ H_k & H_\phi & H_h \end{bmatrix}$$

where each term in the matrix means the partial derivative evaluated at (k^*, ϕ^*, h^*). Explicitly, these are given by the formulas

$$
\begin{array}{lll}
F_k = -\frac{2\delta}{3} & F_\phi = \frac{A^7\gamma_k^4(\gamma-\gamma_k)^3\pi^3D^2}{\beta^2\delta^3\sigma^3} & F_h = \frac{A^6\gamma_k^3(\gamma-\gamma_k)^3\pi^3D^3}{\beta^4\delta^3\sigma^3} \\
G_k = \frac{A^4\pi^3(\gamma-\gamma_k)^3\gamma_kD^2}{3\beta^2\sigma^2\delta} & G_\phi = -\frac{A^{11}\pi^6\gamma_k^5(\gamma-\gamma_k)^6D^4}{\beta^4\delta^5\sigma^5} & G_h = \frac{2A^{16}\pi^9(\gamma-\gamma_k)^9\gamma_k^7D^5}{3\beta^5\delta^7\sigma^8} \\
H_k = \frac{\beta^4\delta^6\sigma^6}{3A^{12}\gamma_k^6(\gamma-\gamma_k)^6\pi^6D^3} & H_\phi = 0 & H_h = -\beta
\end{array}
$$

Computing the characteristic polynomial and eigenvalues of the Jacobian matrix in general, even using computational software such as Maple, does not produce a result that can be intelligently analyzed. A more feasible approach in this case is to give explicit values to the parameters in the problem, which are chosen so as to be economically sensible, and then solve the resulting system numerically using Maple. Before proceeding, I should say what is meant by "economically sensible" and why the parameters I choose could reasonably be described as such. First and foremost, I require that $\phi^* > 0.98$. As discussed earlier, expected lifetime is determined by the formula $1/(1-\phi^*)$. [1] Since the average life expectancy in rich countries is approximately 80 years, while the average in poorer countries typically ranges between 40 and 60 years, the value for ϕ^* should produce a value for life expectancy somewhere in this range. It turns out that a life expectancy of 80 years corresponds to $\phi = 0.9875$, while a life expectancy of 50 years corresponds to $\phi = 0.98$. So a value for ϕ larger than 0.98 (really anywhere between about 0.98 and 0.99) seems like a reasonable estimate for life expectancy. Now in terms of the parameters themselves, a few comments can be made for some of them. Thinking back to my discussion of the success rates for R&D in the prescription drug industry from the introduction, I assume the probability that R&D is effective to be about 25%. My choices for the investment rates give an overall investment rate of 7.5%, which seems to be a sensible estimate for some developing countries (of course

[1] This can be seen most easily in discrete time as follows. Since time is discrete, and ϕ represents the probability of living at each time period, a person can expect to live

$$1 + \phi + \phi^2 + \phi^3 + \ldots = \sum_{n=0}^{\infty} \phi^n = \frac{1}{1-\phi}$$

periods. For my purposes, I measure periods in years.

for wealthy countries I would likely choose an investment rate in excess of 20%). For the depreciation rate, I guess that 5% is an appropriate value. I choose a small depreciation value for β, under the assumption that knowledge about health care retains its worthiness for a suitably long period of time before becoming outdated. The values for the constants D and σ are chosen more or less arbitrarily, although I do assume they should not be too large since they play a similar role in the dynamics as the rates of investment and depreciation. Now for a specific example, the set of values

$$(A, \pi, \gamma_k, \gamma_\phi, \delta, D, \sigma, \beta) = \left(1.614, \frac{1}{4}, \frac{1}{20}, \frac{1}{40}, \frac{1}{20}, \frac{1}{10}, \frac{7}{100}, \frac{1}{100}\right)$$

results in the value $\phi \approx 0.981$ with corresponding eigenvalues

$$\lambda_1 = -0.0342318678, \qquad \lambda_2 = -0.0097877163, \qquad \lambda_3 = -0.0000142599$$

Since these eigenvalues are all real and negative, the steady-state solution is an asymptotically stable steady-state of the system. While this does not prove that the steady-state solution will be stable in real-life situations (since my estimates on the parameters are largely just educated guesses), it does give me some hope that this will be the case in a model whose parameters are given by actual economic data.

The comparative statics in this case are relatively straightforward to analyze. For the situation of primary interest to me (when capital's share of income is $\alpha = \frac{1}{3}$), I obtain the following most pleasing result: the relationships between the variables physical capital, life expectancy, and health knowledge, and the four parameters I am considering, are all positive. This implies several nice conclusions. First of all, the positive relationships from my previous model are all maintained, so in extending our model to include knowledge about health care, I have not lost any information or intuitive conclusions that were present in the first model. Further, knowledge about health care is positively related to each of the parameters, which seems to be sensible: any increase in investment, capital, and R&D should eventually provide

new information about health care (or the ability to transmit this information to the public). Perhaps the most satisfactory observation for this model as compared to the previous one is that the non-intuitive result I obtained regarding a negative relationship between physical capital and investment in physical capital is no longer present.

There is one aspect of the model described by the comparative statics that gives cause for concern. Our above discussion was all under the assumption of $\alpha = \frac{1}{3}$, but in fact the same observations and conclusions hold for any value α with $\alpha < \frac{1}{2}(3 - \sqrt{5}) \approx 0.38$. However, if capital's share of income is greater than this value, then all the relationships between the variables and the parameters become negative, an absurd answer. Now in the previous model, this was less of a concern because the value of α where this sort of behavior occurred ($\alpha = \frac{2}{3}$) was significantly larger than any of the empirically observed data. In this case, however, the value is small enough that the question as to whether or not this model can be sensibly applied to a particular nation's economy cannot be definitively answered unless accurate data is available regarding capital's share of income for that country. This is not an especially grave problem since the number of countries whose observed values exceed 0.38 is relatively small, and as noted earlier, it could be the case that the true value for capital's share of income is lower (this hypothesis is supported by the observation that for most countries where more accurate data is available, the observed values are close to $\frac{1}{3}$).

In summary, taking knowledge about health care into consideration gives a more consistent model than when it is excluded, at least for most of the situations I care about. In particular, the paradoxical result present in the first model has been eliminated, and the new results given by the presence of health knowledge fit both our intuition and the existing work on the subject discussed in the introduction. In view of these observations, I feel justified in claiming that this extended model is an improvement over the basic one.

4. A Generalized Two-Dimensional Model

I generalize the two-dimensional model first considered by deriving a new equation for $\dot{k}$ (see the Appendix for some of the details of this derivation). Inspired by the fascinating results of [1] discussed in the Introduction, I make the assumption that longevity will have a negative effect on output per worker (since it will increase total population size), and incorporate this into the model. Throughout, I use the same notation as in Section 1, and let the new parameter n denote the rate of population growth. Using this new equation for $\dot{k}$ and the equation for $\dot{\phi}$ from the first model, I have the system of equations

$$\dot{k} = \gamma_k A\phi k^\alpha - k(\delta + \dot{\phi} + n) \tag{15}$$

$$\dot{\phi} = \pi\phi(\gamma - \gamma_k)Ak^\alpha - \sigma\phi^2 \tag{16}$$

with initial state of the economy given by the conditions $k(0) = k_0$ and $\phi(0) = \phi_0$ for some values k_0 and ϕ_0.

Setting $\dot{k}$ and $\dot{\phi}$ equal to zero gives the equations

$$\gamma_k A(k^*)^\alpha - k^*(\delta + n) = 0 \tag{17}$$

$$\pi\phi^*(\gamma - \gamma_k)Ak^\alpha - \sigma(\phi^*)^2 = 0 \tag{18}$$

Ignoring the trivial solution $(k^*, \phi^*) = (0, 0)$, I can simplify these as

$$A\gamma_k\phi^*(k^*)^{\alpha-1} = \delta + n \tag{19}$$

$$\pi(\gamma - \gamma_k)A(k^*)^\alpha = \sigma\phi^* \tag{20}$$

Solving equation (19) for ϕ^* gives

$$\phi^* = \left(\frac{\delta + n}{A\gamma_k}\right)(k^*)^{1-\alpha} \tag{21}$$

and substituting equation (21) into equation (20) results in the equation

$$\frac{\sigma(\delta+n)}{A\gamma_k}(k^*)^{1-\alpha} = \pi(\gamma-\gamma_k)A(k^*)^{\alpha} \tag{22}$$

Isolating k^* yields

$$(k^*)^{1-2\alpha} = \frac{A^2\pi\gamma_k(\gamma-\gamma_k)}{\sigma(\delta+n)} \tag{23}$$

which implies that the non-trivial steady-state solution for k^* is

$$k^* = \left(\frac{A^2\pi\gamma_k(\gamma-\gamma_k)}{\sigma(\delta+n)}\right)^{\frac{1}{1-2\alpha}} \tag{24}$$

Finally, substituting equation (24) into equation (21) gives the non-trivial steady-state solution for ϕ^* as

$$\phi^* = \left(\frac{\delta+n}{A\gamma_k}\right)\left(\frac{A^2\pi\gamma_k(\gamma-\gamma_k)}{\sigma(\delta+n)}\right)^{\frac{1-\alpha}{1-2\alpha}} \tag{25}$$

which can be simplified algebraically as

$$\phi^* = \left(\frac{A\pi^{1-\alpha}\gamma_k^{\alpha}(\gamma-\gamma_k)^{1-\alpha}}{\sigma^{1-\alpha}(\delta+n)^{\alpha}}\right)^{\frac{1}{1-2\alpha}} \tag{26}$$

So I get a unique non-zero steady-state solution (k^*, ϕ^*), given by

$$(k^*, \phi^*) = \left(\left(\frac{A^2\pi\gamma_k(\gamma-\gamma_k)}{\sigma(\delta+n)}\right)^{\frac{1}{1-2\alpha}}, \left(\frac{A\pi^{1-\alpha}\gamma_k^{\alpha}(\gamma-\gamma_k)^{1-\alpha}}{\sigma^{1-\alpha}(\delta+n)^{\alpha}}\right)^{\frac{1}{1-2\alpha}}\right) \tag{27}$$

and the steady-state output per worker is given by

$$y_{ss} = \left(\frac{A^{1/\alpha}\pi\gamma_k(\gamma-\gamma_k)}{\sigma(\delta+n)}\right)^{\frac{\alpha}{1-2\alpha}}$$

The problem of determining whether or not the steady-state (k^*, ϕ^*) is stable can easily be shown intractable by working through the calculations. While formulas for the partial derivatives can be explicitly worked out (and are given below), the entries of the Jacobian matrix (consisting of these partial derivatives, evaluated at (k^*, ϕ^*)) are quite complicated.

The eigenvalues of this matrix can be obtained using computational software such as Maple, but this is of limited usefulness as the formulas obtained are also far too complicated to analyze for even basic properties, such as whether they are positive or negative (in the event that they can even be shown to be real). My approach then must be to make assumptions on the values of the parameters $\alpha, A, \pi, \gamma, \gamma_k, \sigma, \delta$, and n, and compute the resulting eigenvalues numerically, as was done for the model which included knowledge about health care.

Setting $\dot{k} = F(k, \phi)$ and $\dot{\phi} = G(k, \phi)$, I have the formulas

$$G(k, \phi) = A\pi(\gamma - \gamma_k)\phi k^{\alpha} - \sigma\phi^2 \tag{28}$$

$$F(k, \phi) = A\gamma_k \phi k^{\alpha} - k(\delta + n) - kG(k, \phi) \tag{29}$$

from which I obtain the partial derivatives

$$G_k = \alpha A\pi(\gamma - \gamma_k)\phi k^{\alpha-1} \tag{30}$$

$$G_\phi = A\pi(\gamma - \gamma_k)k^{\alpha} - 2\sigma\phi \tag{31}$$

$$F_k = \alpha A\gamma_k \phi k^{\alpha-1} - \delta - n - A\pi(\gamma - \gamma_k)\phi k^{\alpha} + \sigma\phi^2 - \alpha A\pi(\gamma - \gamma_k)\phi k^{\alpha} \tag{32}$$

$$F_\phi = A\gamma_k k^{\alpha} - k(A\pi(\gamma - \gamma_k)k^{\alpha} - 2\sigma\phi). \tag{33}$$

The corresponding Jacobian matrix is

$$J = \begin{bmatrix} F_k(k^*, \phi^*) & F_\phi(k^*, \phi^*) \\ G_k(k^*, \phi^*) & G_\phi(k^*, \phi^*) \end{bmatrix}$$

As in the case of the three-dimensional model, I make a rough estimate as to sensible choices of the parameters by insisting that $\phi^* > 0.98$, that $\alpha = \frac{1}{3}$, that the other parameters take values as described for the previous model (the reasoning here is the same, so I omit further comment on these and refer back to the discussion in section 3), and that n is relatively small

(since it represents population growth over total population). The choice of parameters

$$(A, \pi, \gamma, \gamma_k, \sigma, \delta, n) = \left(2.81, \frac{1}{4}, \frac{3}{40}, \frac{1}{20}, \frac{1}{50}, \frac{1}{20}, \frac{1}{100}\right)$$

results in $\phi^* \approx 0.985$ and yields the numerical eigenvalues

$$\lambda_1 = -0.0898071590, \qquad \lambda_2 = -0.00804242076$$

Since both eigenvalues are real and negative, it follows that (k^*, ϕ^*) is a stable steady-state for the system under this setup. Once again, I caution that this does not prove the steady-state will be stable under all real-life situations, but rather it suggests this might end up being true. An interesting question for the future would be to more carefully consider what ranges on the parameters make the most sense economically, and among these which ones result in stability of the steady-state solution.

An economic analysis of the mathematical equations given by looking at comparative statics for this model leads to several interesting results, especially when considered in comparison to those for the first two models. Again, I begin by considering the case where capital's share of income is assumed to be $\frac{1}{3}$. For this situation, I observe positive relationships for both physical capital and life expectancy with the parameters of productivity, probability that R&D is effective, and investment rate in R&D. Also, life expectancy has a positive relationship with investment in physical capital. Regarding the new parameter of population growth, I see that both physical capital and life expectancy display negative relationships with it. This confirms that my model is at least partly consistent with the results of Acemoglu and Johnson that motivated the introduction of the population growth variable into the model: increased longevity results in a downward shift in steady-state physical capital. It is perhaps less clear if life expectancy should relate negatively with population growth, although there is some published work that supports related conclusions. For example, the research of Gregson et. al. ([7]) found that life expectancy decreased in Zimbabwe even though population

growth remained roughly constant and positive. A central aspect of their research which I do not consider here explicitly is the role played by the HIV/AIDS epidemic, and it could very well be possible that this needs to be taken into consideration in order to adequately explain the results I have obtained on the relationship between life expectancy and population growth. This will be discussed more fully in the conclusion.

In the basic model, a serious issue that arose was the negative relationship between physical capital and investment in physical capital. For my extended model, this defect has been rectified somewhat, but not entirely. Precisely, in my current situation ($\alpha = \frac{1}{3}$), physical capital is positively related to investment in physical capital if and only if the investment rate in R&D exceeds the investment rate in physical capital. Clearly, this result is not totally satisfactory. In the extended model that included knowledge about health care, I obtained the superior result of always having a positive relationship between these two quantities. Here I lose that exactitude, and also obtain a somewhat counter-intuitive result: the relationship between physical capital and investment in physical capital is positive only so long as the investment rate in physical capital is lower than the investment rate in R&D, rather than higher.

As was the case with the two other models, once capital's share of income exceeds a certain threshold, a significant shift occurs in the behavior of the comparative statics, with many of the relationships changing from positive to negative, or vice versa. However in my new model, this change occurs at the value $\alpha = \frac{1}{2} = 0.5$, which is well on the high end of the measured values for capital's share of income, and not at all close to the mean value for the study to which I have continually referred. As was the case with the second model, when this change occurs the model becomes nonsensical, with negative relationships where positive ones are expected. This should be viewed as an advantage of our generalized two-dimensional model over the three-dimensional model, where the threshold for α was much closer to the mean. While I cannot be confident that our model can be reliably applied to

all the countries I wish to consider (for example, the empirical value for capital's share of income in Botswana, a developing country in Sub-Saharan Africa, is about 0.55), it should at least be applicable to many of them.

To recap, I have developed a model which produces results that are consistent with many of the desirable ones from my previous attempts, and with some of the results of the research that originally motivated our generalization. Moreover, this model improves upon a problem or defect present in each of the other two. Although it does contain an undesirable defect of its own, I feel that this new model is a noticeable improvement over the original one, and is at least as successful as the three-dimensional version, although it is difficult to say if one of these latter two is definitively better than the other.

5. Conclusion

My goal in this paper has been to understand how economic development interacts with investments and improvements in health care. Since this is an active field of research with many already-established results, the standard literature should be a source of motivation for my analysis, and the results which I obtain should agree and complement what has previously been done. This is indeed the case for some of my conclusions. Each model offers insight into the dynamics I am hoping to study, and both of my attempts to extend the basic model further prove to be at least partly successful. All three of my models offer insight into the importance of R&D for both health improvements, and growth in physical capital. In each subsequent model, I offer improvements over these results by including different ways health can affect economic outcomes. With respect to the second model, I observe that knowledge about health care is an important factor for both health improvements and economic growth, and it indeed warrants explicit inclusion into the dynamics. For the third model, my result describing the negative influence of increased longevity on per capita physical capital

is consistent with the work of Acemoglu and Johnson ([1]), which was the motivation for including population growth into my model. While both extensions of the basic model can be viewed as improvements, it is much less clear if either of them is superior to the other. A further generalization, taking into account both the effect of knowledge about health care and the effect of increased life expectancy on per capita wealth, would perhaps capture the best aspects of each model while removing some of the peculiarities and less intuitive results. I leave this line of research open for future study.

Finally, I highlight one other area worthy of future research: the effect of HIV/AIDS on Africa's growth. While there is no doubt over the profound impact this crisis has had and will continue to have on both the life expectancy and overall welfare in these countries, its effect on economic growth is a source of considerable debate. The more popular view is that the HIV/AIDS epidemic will have a serious negative effect on economic performance in Sub-Saharan African countries. For example, Ardnt and Lewis ([2]) have constructed a model to simulate the economic future of South Africa, under both a "no-AIDS" scenario where the economy continues to perform roughly as it had been, and an "AIDS" scenario where HIV/AIDS-related factors are incorporated and do affect factors that are important for economic performance. Among their findings is that in the two scenarios, there is a consistent divergence of the associated GDP growth rates, to the point that after 13 years the GDP for the AIDS scenario is 17 percent lower than the GDP for the no-AIDS scenario. Moreover, they also find that this discrepancy is not merely the result of population decline as a result of increased AIDS-related mortality, as GDP per capita in their model showed a decrease of 8 percent in the AIDS scenario. A minority view, related to the conclusions of [1], is the work of Young ([19]). Young's economic simulation takes into account the effect of the epidemic on fertility rates , where he finds that its negative effect on accumulation of human capital is overpowered by its negative effect of fertility rates. The net result is that per capita resources should actually increase. In light of both the seriousness of the HIV/AIDS epidemic and the debate in the existing literature, perhaps a more suitable model of our type

for Sub-Saharan African countries would be one that considers HIV/AIDS-related factors explicitly in the dynamics, rather than merely including some of them implicitly in more general parameters.

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6. Appendix

I provide some of the mathematical details used to justify my economic analysis of the models. In particular, I provide details of the derivations for some of the dynamical equations which appear in my models. I also explicitly work out the mathematical formulas for the comparative statics that I have considered in each case, and explain why my various conclusions about their signs (positive or negative) are valid. I use the notation and terminology given in the main body of the paper throughout.

For the basic model, I have assumed that production is given by the Cobb-Douglas production function

$$Y = F(k, L) = AK^{\alpha}L^{1-\alpha}$$

with corresponding output per worker given by

$$y = Ak^{\alpha}$$

Recalling my discussion prior to the presentation of the model, I begin with the equation

$$\dot{K} = \gamma_k \phi Y - \delta K$$

which measures the change in physical capital as the output which is being re-invested into the economy less the depreciation of existing capital. With $k = K/L$, and observing that

$$\frac{Y}{L} = \frac{AK^{\alpha}}{L^{\alpha}} = A\left(\frac{K}{L}\right)^{\alpha} = Ak^{\alpha}$$

it follows that the derivative of k with respect to time is

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{K}{L}\right) = \frac{1}{L}\frac{dK}{dt} - \frac{K}{L^2}dLdt$$

or more compactly

$$\dot{k} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L}$$

Since I am ignoring the effect of population growth on capital (this is what I later consider in the extended model), I have $\dot{L} = 0$, and so the equation for capital accumulation is given by

$$\dot{k} = \frac{\dot{K}}{L} = \gamma_k \phi \frac{Y}{L} - \delta \frac{K}{L} = \gamma_k \phi A k^{\alpha} - \delta k$$

Recall that for the basic model, I have

$$k^* = \frac{A^{2/\alpha}\gamma_k^3\sigma^{3-2/\alpha}}{\pi^{3-2/\alpha}\delta^3(\gamma-\gamma_k)^{3-2/\alpha}}, \qquad \phi^* = \frac{A^{1/\alpha}\gamma_k\sigma^{1-1/\alpha}}{\pi^{1-1/\alpha}\delta(\gamma-\gamma_k)^{1-1/\alpha}}$$

I look at some comparative statics by taking the partial derivatives of k^* and ϕ^* with respect to A, π, γ, and γ_k.

$$\begin{aligned}
\frac{\partial k^*}{\partial A} &= \frac{2}{\alpha A}k^* \\
\frac{\partial \phi^*}{\partial A} &= \frac{1}{\alpha A}\phi^* \\
\frac{\partial k^*}{\partial \pi} &= \frac{2-3\alpha}{\alpha\pi}k^* \\
\frac{\partial \phi^*}{\partial \pi} &= \frac{1-\alpha}{\alpha\pi}\phi^* \\
\frac{\partial k^*}{\partial \gamma} &= \frac{2-3\alpha}{\alpha(\gamma-\gamma_k)}k^* \\
\frac{\partial \phi^*}{\partial \gamma} &= \frac{1-\alpha}{\alpha(\gamma-\gamma_k)}\phi^* \\
\frac{\partial k^*}{\partial \gamma_k} &= \left(\frac{3}{\gamma_k} - \frac{2-3\alpha}{\alpha(\gamma-\gamma_k)}\right)k^* \\
\frac{\partial \phi^*}{\partial \gamma_k} &= \left(\frac{1}{\gamma_k} - \frac{(1-\alpha)}{\alpha(\gamma-\gamma_k)}\right)\phi^*
\end{aligned}$$

I make the following observations. Since $k^* \geq 0, \phi^* \geq 0$, and $A > 0$, it follows that k^* and ϕ^* always display a positive relationship with A. Since $0 < \pi \leq 1$, the relation between k^* and π is positive when $\alpha < \frac{2}{3}$, zero when $\alpha = \frac{2}{3}$, and negative when $\alpha > \frac{2}{3}$, while the relation between ϕ^* and π is always positive. The relation between k^* and γ is positive when $\alpha < \frac{2}{3}$, is zero when $\alpha = \frac{2}{3}$, and is negative when $\alpha > \frac{2}{3}$, while the relation between ϕ^* and γ is again always positive. Finally, the relation between k^* and γ_k is positive if $\gamma_k < \frac{3\alpha}{2}\gamma$, negative if $\frac{3\alpha}{2}\gamma < \gamma_k$, and zero if $\gamma_k = \frac{3\alpha}{2}\gamma$, while the relation between ϕ^* and γ_k is positive if $\gamma_k < \alpha\gamma$, negative if $\alpha\gamma < \gamma_k$, and zero if $\gamma_k = \alpha\gamma$. In particular, when $\alpha = \frac{2}{3}$, the relation between k^* and γ_k is zero if and only if $\gamma_k = \gamma$ (that is, no investment goes toward R&D), and is positive otherwise. If $\alpha > \frac{2}{3}$, then $\frac{3\alpha}{2}\gamma > \gamma$, and so the relation between k^* and γ_k is always positive.

For the extended model that includes knowledge about health care, I have assumed that production is given by the Cobb-Douglas production function

$$Y = F(K, hL) = AK^{\alpha}h^{1-\alpha}L^{1-\alpha}$$

with corresponding output per healthy worker given by

$$y = Ak^{\alpha}$$

I again begin with the equation

$$\dot{K} = \gamma_k \phi Y - \delta K$$

which measures the change in physical capital as the output which is being re-invested into the economy less the depreciation of existing capital. With $k = K/hL$, and observing that

$$\frac{Y}{hL} = \frac{AK^{\alpha}}{h^{\alpha}L^{\alpha}} = A\left(\frac{K}{hL}\right)^{\alpha} = Ak^{\alpha}$$

it follows that the derivative of k with respect to time is

$$\frac{dk}{dt} = \frac{d}{dt}\left(\frac{K}{hL}\right) = \frac{1}{hL}\frac{dK}{dt} - \frac{K}{h^2L^2}\left(h\frac{dL}{dt} + L\frac{dh}{dt}\right)$$

or more compactly

$$\dot{k} = \frac{\dot{K}}{hL} - \frac{K}{hL}\frac{\dot{h}}{h}$$

Since I am ignoring the effect of population growth on capital (this is what I later consider in the extended model), I have $\dot{L} = 0$, and so the equation for capital accumulation is given by

$$\dot{k} = \frac{\dot{K}}{hL} - \frac{K}{hL}\frac{\dot{h}}{h} = \gamma_k \phi \frac{Y}{hL} - \delta\frac{K}{hL} - \frac{K}{hL}\dot{h} = \gamma_k \phi A k^{\alpha} - k(\dot{h} + \delta)$$

Now I can look at comparative statics by taking the partial derivatives of k^*, ϕ^*, and h^* with respect to A, π, γ_k, and γ_ϕ:

$$\frac{\partial k^*}{\partial A} = \frac{2}{(\alpha^2 - 3\alpha + 1)A}k^*$$

$$\frac{\partial \phi^*}{\partial A} = \frac{1 + \alpha - \alpha^2}{(\alpha^2 - 3\alpha + 1)A}\phi^*$$

$$\frac{\partial h^*}{\partial A} = \frac{2\alpha}{(\alpha^2 - 3\alpha + 1)A}h^*$$

$$\frac{\partial k^*}{\partial \pi} = \frac{1}{(\alpha^2 - 3\alpha + 1)\pi}k^*$$

$$
\begin{aligned}
\frac{\partial \phi^*}{\partial \pi} &= \frac{1-\alpha}{(\alpha^2-3\alpha+1)\pi}\phi^* \\
\frac{\partial h^*}{\partial \pi} &= \frac{\alpha}{(\alpha^2-3\alpha+1)\pi}h^* \\
\frac{\partial k^*}{\partial \gamma_k} &= \frac{1}{(\alpha^2-3\alpha+1)\gamma_k}k^* \\
\frac{\partial \phi^*}{\partial \gamma_k} &= \frac{\alpha(2-\alpha)}{(\alpha^2-3\alpha+1)\gamma_k}\phi^* \\
\frac{\partial h^*}{\partial \gamma_k} &= \frac{\alpha}{(\alpha^2-3\alpha+1)\gamma_k}h^* \\
\frac{\partial k^*}{\partial \gamma_\phi} &= \frac{1}{(\alpha^2-3\alpha+1)\gamma_\phi}k^* \\
\frac{\partial \phi^*}{\partial \gamma_\phi} &= \frac{1-\alpha}{(\alpha^2-3\alpha+1)\gamma_\phi}\phi^* \\
\frac{\partial h^*}{\partial \gamma_\phi} &= \frac{\alpha}{(\alpha^2-3\alpha+1)\gamma_\phi}h^*
\end{aligned}
$$

Observing that for $0 < \alpha < 1$, all the terms $A, \pi, \gamma_k, \gamma_\phi, k^*, \phi^*, h^*, \alpha, 1-\alpha, 2-\alpha$, and $1+\alpha-\alpha^2$ are non-negative, it follows that relationships between k^*, ϕ^*, and h^*, and the parameters A, π, γ_k, and γ_ϕ, are positive for $0 < \alpha < \frac{1}{2}(3-\sqrt{5})$, and are negative for $\frac{1}{2}(3-\sqrt{5}) < \alpha < 1$. So in this model, the relationships between k^*, ϕ^*, h^* and the parameters demonstrate a very strong dependence on the value of α, with the change happening at $\alpha = \frac{1}{2}(3-\sqrt{5}) \approx 0.38$.

The derivation for the $\dot{k}$ equation in the generalized two-dimensional model proceeds as follows. I begin with the same equation as before to describe the dynamics of K, namely

$$\dot{K} = \gamma_k \phi Y - \delta K$$

where again I am measuring output with

$$Y = AK^\alpha L^{1-\alpha}$$

I also have the familiar equation for output per worker given by

$$y = \frac{Y}{L} = A\frac{K^\alpha}{L^\alpha} = A\left(\frac{K}{L}\right)^\alpha = Ak^\alpha$$

where $k = K/L$. The derivative of k with respect to time is

$$\frac{dk}{dt} = \frac{1}{L}\frac{dK}{t} - \frac{K}{L^2}\frac{dL}{dt}$$

Now the key change from the basic model takes place. Rather than assuming that $\dot{L} = 0$ as previously done, I consider the effect of population growth. Let $P = P(t)$ denote the population at time t which can contribute to the labor force. I assume that $L = \phi P$, and $n = \frac{\dot{P}}{P}$ is the rate of population growth. If I approximate slightly by assuming that time is discrete, I can write

$$L(t) = \phi(t)P(t) = \phi(t)(1+n)P(t-1)$$

which implies that

$$\frac{L(t+1)}{L(t)} = \frac{\phi(t+1)P(t+1)}{\phi(t)P(t)} = \frac{\phi(t+1)}{\phi(t)}(n+1)$$

Now thinking of time as changing continuously, I see that the correct equation is

$$\frac{\dot{L}}{L} = \dot{\phi} + n$$

This implies

$$\begin{aligned}
\dot{k} &= \frac{1}{L}\dot{K} - \frac{K}{L} \cdot \frac{1}{L}\dot{L} \\
&= \frac{1}{L}\dot{K} - k(\dot{\phi} + n) \\
&= \frac{1}{L}\dot{K} - k(\dot{\phi} + n) \\
&= \frac{\dot{K}}{L} - k(\dot{\phi} + n)
\end{aligned}$$

So I have

$$\dot{k} = (\gamma_k \phi A k^\alpha - \delta k) - k(\dot{\phi} + n)$$

which I can re-write as

$$\dot{k} = \gamma_k \phi A k^\alpha - k(\delta + \dot{\phi} + n)$$

Comparative statics for the generalized two-dimensional model are not difficult to compute, and are seen to have a relatively simple mathematical behavior. Writing

$$k^* = \left(\frac{A^2 \pi \gamma_k (\gamma - \gamma_k)}{\sigma(\delta + n)}\right)^{\frac{1}{1-2\alpha}}$$

and

$$\phi^* = \left(\frac{A \pi^{1-\alpha} \gamma_k^\alpha (\gamma - \gamma_k)^{1-\alpha}}{\sigma^{1-\alpha}(\delta + n)^\alpha}\right)^{\frac{1}{1-2\alpha}}$$

I obtain the following partial derivative formulas:

$$
\begin{aligned}
\frac{\partial k^*}{\partial A} &= \frac{2}{(1-2\alpha)A}k^* \\
\frac{\partial \phi^*}{\partial A} &= \frac{1}{(1-2\alpha)A}\phi^* \\
\frac{\partial k^*}{\partial \pi} &= \frac{1}{(1-2\alpha)\pi}k^* \\
\frac{\partial \phi^*}{\partial \pi} &= \frac{1-\alpha}{(1-2\alpha)\pi}\phi^* \\
\frac{\partial k^*}{\partial \gamma} &= \frac{1}{(1-2\alpha)(\gamma-\gamma_k)}k^* \\
\frac{\partial \phi^*}{\partial \gamma} &= \frac{1-\alpha}{(1-2\alpha)(\gamma-\gamma_k)}\phi^* \\
\frac{\partial k^*}{\partial \gamma_k} &= \left(\frac{1}{(1-2\alpha)\gamma_k} - \frac{1}{(1-2\alpha)(\gamma-\gamma_k)}\right)k^* \\
\frac{\partial \phi^*}{\partial \gamma_k} &= \left(\frac{\alpha}{(1-2\alpha)\gamma_k} + \frac{\alpha-1}{(1-2\alpha)(\gamma-\gamma_k)}\right)\phi^* \\
\frac{\partial k^*}{\partial n} &= \frac{-1}{(1-2\alpha)(\delta+n)}k^* \\
\frac{\partial \phi^*}{\partial n} &= \frac{-\alpha}{(1-2\alpha)(\delta+n)}\phi^*
\end{aligned}
$$

The analysis of these equations is relatively straightforward, with two exceptions. I first observe that the value $\alpha = \frac{1}{2}$ must be omitted. Since $0 < \alpha < 1$, the only quantity whose sign can change as α changes is $1 - 2\alpha$. It follows that the partial derivatives of k^* with respect to A, π, and γ are positive for $\alpha < \frac{1}{2}$ and negative for $\alpha > \frac{1}{2}$. Similarly, the partial derivatives of k^* with respect to γ_k and n are negative for $\alpha < \frac{1}{2}$ and positive for $\alpha > \frac{1}{2}$. For ϕ^*, I see that the partial derivatives with respect to A, π, and γ are positive when $\alpha < \frac{1}{2}$ and negative when $\alpha > \frac{1}{2}$, and that the partial derivatives with respect to n is negative when $\alpha < \frac{1}{2}$ and positive when $\alpha > \frac{1}{2}$. The only equations whose behaviors are somewhat complicated are the partial derivatives of k^* and ϕ^* with respect to γ_k, which depend on both the value of α as well as the relationship between α, γ and γ_k. In particular, I can say

the following:

$$\frac{\partial k^*}{\partial \gamma_k} = \begin{cases} \text{positive} & \text{if} \quad \alpha < \frac{1}{2} \quad \text{and} \quad 2\gamma_k < \gamma \\ \text{positive} & \text{if} \quad \alpha > \frac{1}{2} \quad \text{and} \quad 2\gamma_k > \gamma \\ \text{negative} & \text{if} \quad \alpha < \frac{1}{2} \quad \text{and} \quad 2\gamma_k > \gamma \\ \text{negative} & \text{if} \quad \alpha > \frac{1}{2} \quad \text{and} \quad 2\gamma_k < \gamma \\ 0 & \text{if} \quad \alpha \neq \frac{1}{2} \quad \text{and} \quad \gamma = 2\gamma_k \end{cases}$$

and

$$\frac{\partial \phi^*}{\partial \gamma_k} = \begin{cases} \text{positive} & \text{if} \quad \frac{\gamma_k}{\gamma} < \alpha < \frac{1}{2} \quad \text{or} \quad \frac{1}{2} < \alpha < \frac{\gamma_k}{\gamma} \\ \text{negative} & \text{if} \quad \alpha < \min\left\{\frac{1}{2}, \frac{\gamma_k}{\gamma}\right\} \quad \text{or} \quad \alpha > \max\left\{\frac{1}{2}, \frac{\gamma_k}{\gamma}\right\} \\ 0 & \text{if} \quad \alpha = \frac{\gamma_k}{\gamma} \neq \frac{1}{2} \end{cases}$$

So a significant change in the behavior of the system occurs at the transition point $\alpha = \frac{1}{2}$, namely everything that was increasing starts to decrease and everything that was decreasing starts to increase (with one exception, as shown above).